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# NATIONAL SCHOOL OF BANKING

## ALGEBRA

Algebra is one of the branches of mathematics just like arithmetic, geometry, statistics etc. In algebra, letters are used to represent numbers.

Letters which are used to represent numbers are called littorals. Using littorals in place of numbers helps us to think in a more general way and obtain a rule. This enables us to generalise often repeated numerical statements. That is why algebra is called generalised arithmetic.

Basic algebra deals with methods of doing arithmetic calculations like addition, subtraction, multiplications etc. with literal numbers.

Since the letter x is used frequently in algebra, it is confusing to use the ordinary multiplication symbol 'x' as it represents x. The product of two or more literal numbers of a numerical number and one or more literal numbers is written without the ordinary multiplication symbol.

a b is written as ab

7 x is written as 7x

x 7 is written as 7x and not as x7 by convention.

This symbolism cannot be adopted for numericals. 5 6 cannot be written as 56, because 56 has a different meaning. The product of 5 and 6 has to be written as 5 6 only.

**VARIABLES AND CONSTANTS** :

In arithmetic, numbers have a definite meaning. The value that a number has does not change.
But letters used in algebra have no particular value and can have any value assigned to them.

For example - The perimeter of a rectangle of length l and breadth 'b' is given as -

$$P = 2 (l + b).$$

Here 2 is a fixed number. The letters P, l and b have no fixed value assigned to them. They can take any positive value, depending upon the size of the rectangle.

- A letter symbol which can take any value is called as a variable.
- Quantities which have only one fixed value are called constants.

**NOTE** :

In some cases, literal numbers are also used as constants. In such cases, the fact will be specifically mentioned.

**TERM** :

Numerical numbers alone, or literal numbers alone or their combinations by operations of multiplication are called terms.

Thus, $3, -7, \frac{2}{5}, x, ab, 5m$ etc. are terms.`

**ALGEBRAIC EXPRESSION** :

The combinations of terms obtained by the operations of '+' or '-' or both is called an algebraic expression.

Thus, $6y + 4z$, $4a - 2b$ and $8p - 7q + \frac{3}{8} r$ are all algebraic expressions.

Expressions are classified by the number of terms they contain. Thus,

- An expression containing only one term is called a monomial. e.g. 7x, 6y, $\frac{5z}{3x}$ etc.
- An expression containing two terms is called as binomial. e.g. 8x + 3, 6y - z, $\frac{w}{2} + 5$
- An expression containing three terms is called as trinomial. e.g. (7x + 4y - 2z), $(3p + \frac{4q}{8} - \frac{3}{r})$ ........
- An expression containing more than 3 terms is called a polynomial or multinomial.

**COEFFICIENT :**

In a product containing two or three or more factors, each factor is called the coefficient of the product of other factors.

For example, in the product of 6xy,

6 is the coefficient of xy,

x is the coefficient of 6y and

y is the coefficient of 6x.

6 is called the numerical coefficient, whereas ‘x’ and ‘y’ are called literal coefficients.

**LIKE AND UNLIKE TERMS :**

Terms which contain the same variables or literal factors are called like terms.

Otherwise, they are called unlike terms.

e.g. 5x and 6x have some literal factors ‘x’, hence they are called like terms.

**Value of an Algebraic Expression :**

The literal numbers or variables can have any value allotted to them. Thus, by substituting the allotted numerical value in place of the variables, the value of an algebraic expression can be found.

## FUNDAMENTAL OPERATIONS IN ALGEBRA

1. **Addition :**

Only like terms can be combined but not the unlike terms.

The sum (or difference) of like terms can be defined as a like term similar to each one of them, whose coefficient is equal to the sum (or difference) of the coefficients of the given terms.

(OR) while adding two like terms, we first add their numerical coefficients and multiply the variables with the sum obtained.

e.g.8x + 26x = (8 + 26) x = 34x. $4x^2y + x^2y = (4 + 1) x^2y = 5x^2y$.

2. **Subtraction :**

To subtract one expression from a 2nd expression, we first change the sign of each term of that (1st) expression and add them with the 2nd expression.

e.g.Subtract $(4x^2 - 6x + 5)$ from $6x^2 + 7x - 9$.

**Step 1 :** Change the sign of each term of 1st expression i.e. it becomes $-4x^2 + 6x - 5$.

Now add it with the 2nd expression.

i.e. $(-4x^2 + 6x - 5) + (6x^2 + 7x - 9)$

$-4x^2 + 6x - 5 + 6x^2 + 7x - 9$

$x^2 (-4 + 6) + x (6 + 7) + (-5 - 9)$

$2x^2 + 13x - 14$.

**Multiplication :**

**To multiply two monomials :**

1) The numerals are multiplied together
2) Literals are multiplied in alphabetical order
3) If signs are same, affix the positive sign or '+' sign to the product.
4) If signs are different for the two monomials, affix the negative or '-' sign to the monomials. e.g.

   $12xy \quad 3z = (12 \quad 3) (xy \quad z) = 36\ xyz$.
5) When we have repeated factors multiplied together, we can infer by observation that the exponents of the repeated factors are actually added.

   e.g. $a^3 \quad a^2 = (a \quad a \quad a) \quad (a \quad a) = a^5$

   $(3a^2b) \quad (-4a^3b^5) = -(3 \quad a^2 \quad a^3 \quad b \quad b^5) \quad 4$

   $= -(12a^5b^6)$

**To multiply a monomial and a binomial :**

We use distributive property i.e. a (b + c) = a b + a c i.e. We multiply each term of the binomial by the monomial and add both the results.

e.g. $3x^2z^3 \quad (2y^2 - 6xz)$

$= 3 \quad 2 \quad x^2 \quad y^2 \quad z^3 - 3 \quad 6 \quad x^2 \quad x \quad z^3 \quad z$

$= 6x^2y^2z^3 - 18x^3z^4$

**To multiply a monomial and a multinomial :**

The distributive property is true when there are more than two terms within the brackets. i.e.

a (b + c + d + ....... g) = a.b + a.c + ....... + a.g.

(b + c + d + ....... + g) a = a.b + a.c + a.d + ........ + a.g.

**To multiply two polynomials :**

Multiply each term of a multinomial with the entire other expression and add the results thus obtained.

(a + b + c + d) (e + f + g + h)

= a (e + f + g + h) + b (e + f + g + h) + c (e + f + g + h) + d (e + f + g + h) e.g. $(3x^2y^2 + 5x^2y^2z^2 + 6) (7a^2b - 6a^2c - 8cb^2)$

$= 3x^2y^2 \quad (7a^2b - 6a^2c - 8cb^2) + 5x^2y^2z^2 \quad (7a^2b - 6a^2c - 8cb^2) + 6 (7a^2b - 6a^2c - 8cb^2)$

$= 21x^2y^2a^2b - 18x^2y^2a^2c - 24x^2y^2cb^2 + 35x^2y^2z^2a^2b - 30x^2y^2z^2a^2c - 40x^2y^2z^2cb^2 + 42a^2b - 36a^2c - 48cb^2$

**Division :**

In arithmetic, we divide 37 by 5 as -

| | | | |
|---|---|---|---|
| 5) 37 (7 | | 37 | dividend |
| 35 | where | 5 | divisor |
| 2 | | 2 | remainder and |
| | | 7 | quotient |

a) **Division of a monomial by another monomial.** There will be only one term in the numerator and denominator. Hence in this case, division is only to reduce a fraction to its lowest term.

e.g. $36x^3 \quad 9x^2 = \frac{36x^3}{9x^2} = 4x.$

b) **Division of a multinomial by a monomial.** In this case, each term of the multinomial should be divided by the monomial and the results added.

e.g. $(15a^3 \quad + 9a^2 + 6a + 21) \quad 3a$

$$= \frac{15a^3}{3a} + \frac{9a^2}{3a} + \frac{6a}{3a} + \frac{21}{3a} = (5a^2 + 3a + 2 + \frac{21}{3a}).$$

c) **Division of a multinomial by another multinomial :**

Only a multinomial of a higher degree has to be divided by a multinomial of a lower degree.

While dividing, the following steps have to be followed :

1. Arrange the terms of the dividend and the divisor in decreasing order of power.
2. Divide the 1st term of the dividend by the 1st term of the divisor and write the result as the 1st term of the quotient.
3. Multiply the entire divisor by this 1st term of the quotient and put the product under the dividend.
4. Subtract the product from the dividend and bring down the remaining term(s).
5. Step 4 gives the new dividend. Repeat Steps 1 to 4.
6. Continue division till the remainder becomes zero or the degree of the remainder becomes less than that of the divisor.

e.g. divide $(12a^2 - 12ab - 9b^2)$ by $(6a + 3b)$

$6a + 3b)\ 12a^2 - 12ab - ab^2\ ($

1. $\frac{\text{1st term of dividend}}{\text{1st term of divisor}} = \frac{12a^2}{6a} = 2a.$

   1st term of quotient = 2a.

2. $6a + 3b)\ 12a^2 - 12ab - 9b^2\ (2a$

   $12a^2 \pm 6ab$

   $-$

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   $- 18ab - 9b^2$

   Now, the new dividend is $-18ab - 9b^2$

   $\frac{-18ab}{6a} = -3b.$

$$6a + 3b)\ 12a^2 - 12ab - 9b^2\ (2a - 3b$$
$$+ 12a^2 + 6ab$$
$$--$$
$$- 18ab - 9b^2$$
$$- 18ab - 9b^2$$
$$++$$
$$0$$

$(12a^2 - 12ab - 9b^2)\ \ (6a + 3b) = (2a - 3b).$

## EXPANSIONS

The formulae to be remembered are :

$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$

$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

$(x + a)(x + b) = x^2 + ax + bx + ab$

$(x + a)(x - b) = x^2 + ax - bx - ab$

$(x - a)(x + b) = x^2 - ax + bx - ab$

$(x - a)(x - b) = x^2 - ax - bx + ab$

$(a + b + c)^2 = (a + b + c)(a + b + c)$

$= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2$

$= a^2 + b^2 + c^2 + 2(ab + bc + ca).$

$(a + b)^3 = (a + b)(a + b)(a + b)$

$= a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + 3ab(a + b) + b^3$

$(a - b)^3 = (a - b)(a - b)(a - b)$

$= a^3 - 3a^2b + 3ab^2 - b^3$

$= a^3 - 3ab(a - b) - b^3.$

**FACTORISATION :**

If two or more expressions are multiplied together, their product is obtained. Each of the algebraic expressions which are multiplied to term the product are called the factors (divisors) of the product.

e. g. $(x + a)(x + b) = x^2 + ax + bx + ab$

$(x + a)$ and $(x + b)$ are called factors of $x^2 + ax + bx + ab$.

**Highest Common Factor (H.C.F.) :**

H.C.F. of two monomials is the largest monomial, which divides (is a factor of) completely each of the given monomials.

**To find H.C.F. :**

1. Break down each monomial into simplest numerical and literal factors.
2. Find all the common numerical and literal factors that divide each of the given monomials.
3. Multiply such factors to get the H.C.F. (or G.C.D.).